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Acoustic approximation in the slamming problem

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The plane unsteady problem of the deflection of a solid, slightly curved plate in collision with an ideal weakly compressible liquid is considered. In order to describe the impact process, the acoustic approximation and the method of normal modes are used. The analysis is focused on the supersonic stage of the impact when the liquid surface remains undisturbed outside the contact spot between the solid plate and the liquid. However, the positions of the contact points are unknown in advance, in contrast to the case of undeformable body impact, and have to be found together with the liquid flow, the pressure distribution, and the bottom deformations. It was shown that the duration of the supersonic stage depends on the entering body elasticity. A spray jet is formed earlier and the stage at which the liquid compressibility is a governing factor is shorter than under rigid-body impact. It is revealed that the elastic plate deflection is quite small and can be satisfactorily approximated by a few modes. On the other hand, the calculation of the bending stress distribution needs a much greater number of normal modes. The pressure distribution over the contact region is quite difficult to find by the mode method; an alternative approach is suggested.

1. Introduction

The plane unsteady problem of a smooth blunt body penetrating an ideal and weakly compressible liquid is considered. The body bottom is an elastic curved plate and the sidewalls of the body are undeformable. Initially the liquid is at rest and occupies a lower half-plane ($y' < 0$), and the body bottom touches its free surface ($y' = 0$) at a single point (figure 1*a*) taken as the origin of the Cartesian coordinate system $x'Oy'$ (dimensional variables are denoted by a prime). At the initial instant of time ($t' = 0$) the body starts to penetrate the liquid vertically with a constant velocity V. The body velocity is assumed to be much less than the sound velocity in the resting liquid c_0. The shape of the entering body bottom is changed owing to its interaction with the liquid (figure 1*b*). The influence of the air on the process as well as both external mass forces and surface tension are neglected. The presence of the contact points between the free surface and the elastic plate is the main feature of the problem. The positions of these points are unknown in advance and must be determined together with the liquid flow, the pressure distribution, and the bottom deformations.

We shall determine the elastic bottom deflection, the bending stress distribution and the pressure over the contact region, as well as the contact point position under the following assumptions: (i) the bottom of the entering body is solid, elastic, shallow

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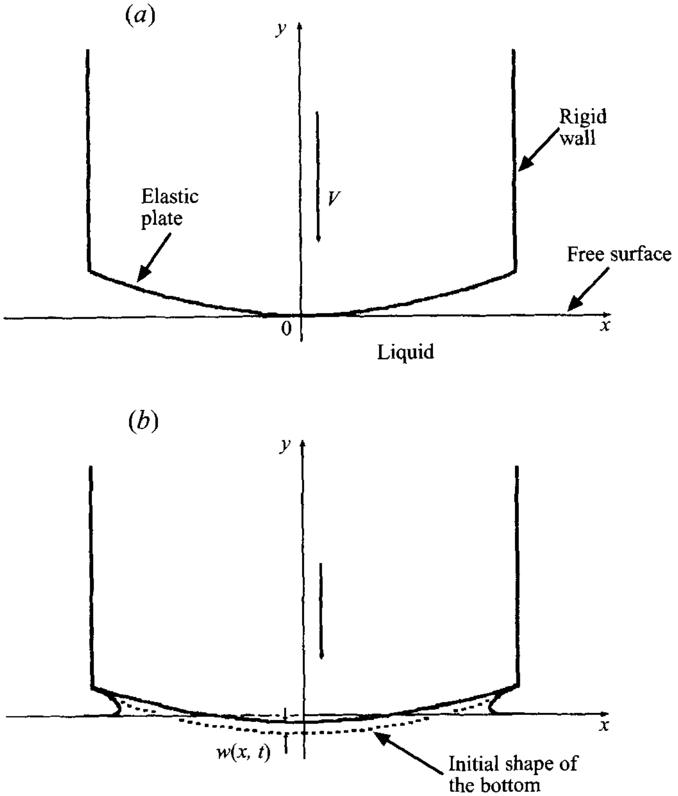

FIGURE 1. Impact of a curved beam on a liquid free surface. Initially, liquid is at rest and occupies a lower half-plane $y < 0$, and an elastic plate touches the free surface at a single point. (*b*) The flow pattern after the impact.

and symmetrical with respect to the y-axis; (ii) the initial radius of the curvature of the top of the plate R is much larger and the bottom plate thickness is much smaller than the plate length $2L$; (iii) the dynamics of the bottom plate deflection is governed by the Euler beam equation; (iv) the bottom plate is fixed at the end points and is connected to the rigid sidewalls by springs; (v) the Mach number $M = V/c_0$ is much less than unity; (vi) the liquid is an acoustic medium, its motion is plane and symmetrical with respect to the y-axis.

It is well-known (Bowden & Field 1964) that within the framework of the above-mentioned assumptions there is an instant T' such that for $0 < t' < T'$ the free surface remains undisturbed (supersonic stage). The presence of this stage is connected with the fact that the expansion velocity of the contact spot for small times is greater than the local sound velocity. In this case, the disturbance front is attached obliquely to the contact points and the disturbed part of the liquid is bounded by the body surface on one side and by the shock wave on the other side. The value of T' can be found from purely geometrical considerations for an undeformable body (see Bowden & Field 1964), but for an elastic body it is unknown in advance and has to be calculated. The quantity T' is a very important parameter of the process: it indicates the duration of the initial stage at which the liquid compressibility is of major significance. Later on, $t' > T'$, the shock front breaks away from the contact points and escapes onto the free surface. After escaping the shock wave (the subsonic stage) relief waves are formed and spread along the contact region and into the liquid bulk. Spray jets appear at this stage. The effects due to the free surface presence are very strong in water impact processes. In order to analyse the combine influence of both liquid compressibility and plate elasticity on the impact, the supersonic stage is considered here.

The problem considered can be transformed by a suitable transformation of the coordinates to the problem of a horizontal elastic plate impact onto the top of a water wave (figure 2). This problem is of great interest for the ship industry, because it is closely connected with wave impact onto the wetdeck of a catamaran (wetdeck slamming). It was intensively studied by Kvålsvold & Faltinsen (1993*a,b*) within the framework of the ideal incompressible liquid model. As pointed out by Korobkin (1992*b*), the above-mentioned coordinate transformation is approximately identical at the initial stage of the impact. Therefore, the present results are valid not only for the problem under consideration but for the problem of slamming against a wetdeck as well. One needs only to calculate the corresponding value of the initial bottom curvature R using known parameters of the wave. All other both geometrical and dynamical characteristics remain the same.

There are many approaches to this problem, which range from quite simple ones as, for example, suggested by Sharov (1958) to complicated ones which can only be analysed with modern powerful computers. The main characteristics of the problem are connected with the fact that the plate deflection is caused by the hydrodynamic load, the region of action of which expands with time and the amplitude of which is dependent of this deflection itself. Thus, the problem under investigation is a coupled problem: the liquid motion and the body deformation, as well as the dimension of the contact region, must be determined simultaneously.

However, just after the impact moment these two parts of the problem can be separated (see Korobkin 1985): first, the solid surface is taken as an undeformable one and the liquid flow can be found, and then the surface deformations caused by the given load are determined. It was shown that in the ideal incompressible liquid model the elastic deflection is of $O([t'V/R]^{3/2})$ and both the velocity of the deformation and the stresses are of $O([t'V/R]^{1/2})$ as $t'V/R \to 0$. Expressions for the deformations were given by Korobkin & Pukhnachov (1985); only the problem for the ideal, weakly compressible liquid model was formulated there. It was revealed that both the liquid flow and the beam deflection are self-similar in time. The deflection depends not on the variables x', t' separately, but on their combination $x'/(RVt')^{1/2}$. Self-similar solutions of these kinds can be used as the initial data to start numerical calculations which are not able in principle to describe the initial stage of the impact when the geometry of the process changes very quickly. The self-similar solutions can also be used to improve our understanding of the impact processes and to test other methods.

On the other hand, it was demonstrated by Kvålsvold & Faltinsen (1993*a*) that the problem solution is stable and weakly depends on variations of the initial data. In order to start the numerical calculations within the framework of the ideal incompressible liquid model, they put the pressure to be constant over the contact spot at the initial stage. It was shown that even large variations of this constant do not effect the solution for quite moderate times. This means that for numerical investigation of the problem we can assume that initially the body bottom contacts the liquid not at a single point as mentioned above but over a spot of a non-zero but quite small dimension. This approach makes the impact problem more suitable for numerical analysis and does not affect significantly the final results.

The goal of the present paper is to demonstrate that the acoustic theory of liquid–solid impact can be generalized to account for the body elasticity. The influence of the entering body elasticity on the both geometrical and dynamical

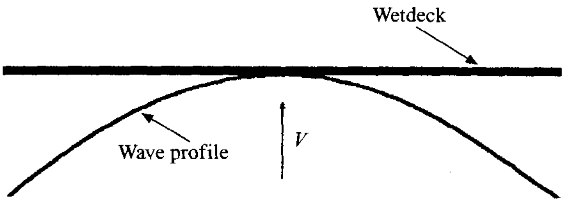

FIGURE 2. Impact of the wetdeck of a catamaran on the top of a water wave.

characteristics of the impact is analysed in detail. Some results are presented in dimensional variables; the properties of the elastic plating and the liquid were taken the same as in the theoretical analysis by Kvålsvold & Faltinsen (1993*a,b*), which is devoted to the slamming against the wetdeck of a catamaran.

The beam is assumed shallow, its initial shape is approximated as parabolic $y' = x'^2/(2R)$ where R is the radius of the curvature at the top of the contour, $L/R \ll 1$. The elastic plate deflection $w'(x', t')$ is governed by the Euler beam equation in which the initial shape of the beam is not taken into account. The position of the plate at the moment t' is given by $y' = x'^2/(2R) + w'(x', t') - Vt'$. In order to estimate the duration of the supersonic stage T', we neglect the elastic deflections of the beam and take into account that the liquid free surface in the time interval $0 < t' < T'$ remains undisturbed, $y' = 0$. The coordinate $x_i'(t')$ of the right-hand contact point between the rigid surface, $w' \equiv 0$, and the liquid free boundary satisfies the equation $0 = x_i'^2/(2R) - Vt'$, which gives $x_i'(t') = (2RVt')^{1/2}$. The speed of the contact region expansion $\mathrm{d}x_i'/\mathrm{d}t' = (RV/2t')^{1/2}$ is much greater just after the impact moment than the local speed of signal propagation and vanishes with time. The sound velocity in liquid can be approximated by constant c_0 for $M \ll 1$. Therefore, the disturbance front generated under the impact escapes onto the free surface at the moment T' when $(\mathrm{d}x_i'/\mathrm{d}t')(T') = c_0$. That gives $T' = \frac{1}{2}(R/V)M^2$. We assume that the elastic effects cannot change the order of the supersonic stage duration, and take the quantity $(R/V)M^2$ as the time scale in the problem considered here. Also, $x_i'(T') = RM$ is taken as the length scale. The supersonic stage is of major importance when the length scale introduced is of order of the beam length, $RM = O(L)$, which leads to the condition $R = O(Lc_0/V)$. This condition implies that the liquid compressibility has to be taken into account under the impact of a very shallow plating onto the horizontal water surface and under impact of long waves onto the wetdeck of a catamaran (see §6).

At the supersonic stage of the impact concerned the deformations of the liquid volume are infinitesimal, which allows us, as a first approximation, to put the boundary conditions on the undisturbed initial level of the liquid and to linearize them and the equations of motion near the initial rest state. The linearization leads to the acoustic approximation where the liquid flow is irrotational and is described by the velocity potential. The potential satisfies the wave equation in the lower half-plane ($y' < 0$). Its normal derivative on the line $y' = 0$ is equal in the contact region to the vertical velocity of the elastic body, which is the sum of the impact velocity V and the local velocity of the beam deflection $\mathrm{d}w'/\mathrm{d}t'$, and is equal to zero on the free surface, which remains undisturbed at the stage under consideration here. Therefore, it is convenient to take the entry velocity V as the velocity scale of the flow. This is the supersonic stage, at which the total hydrodynamic force on the entering plate takes its maximum, and the pressure in the contact region can drop below its initial atmospheric value due to elastic effects only.

2. Formulation of the problem

The plane problem of the impact of a shallow beam onto the surface of an acoustic medium is considered in non-dimensional variables. The scales of the independent variables are chosen the same as in the rigid-body impact problem (Korobkin 1992*a*). They are: RM is the length scale, RM^2 is the liquid displacement scale, M^2R/V is the time scale, V is the velocity scale of the liquid flow, $\rho_0 c_0 V$ is the pressure scale, where ρ_0 is the liquid density. The scale of the beam deflection is taken to be $\rho_0 R^2 M^3/M_B$ where M_B is the beam mass per unit length. The scale of the bending stresses in the beam is equal to $\rho_0 EMh/M_B$ where E is the elasticity modulus, h is the beam thickness. The choice of the time, length and velocity scales was explained in §1. Other scales are introduced so that the sound velocity, the impact velocity and the beam mass per unit length are equal to unity in the non-dimensional variables.

The liquid motion is governed by the wave equation for the velocity potential $\phi(x,y,t)$, and the bottom deformation by the Euler beam equation for the deflection $w(x,t)$. In the first approximation the conditions on the liquid boundary can be linearized and taken on the undisturbed liquid level. The positions of the contact points are described in the symmetrical case by the function $c(t)$. Despite the fact that both the equations of motion and the boundary conditions are linearized, the problem remains nonlinear as $c(t)$ is unknown. The coupled problem has the form

$$\left.\begin{aligned}
&\phi_{tt} = \phi_{xx} + \phi_{yy} \quad (y < 0),\\
&\phi_y = -1 + \kappa w_t(x,t) \quad (y = 0, |x| < c(t)),\\
&\phi = 0 \quad (y = 0, |x| > c(t)),\\
&\phi \to 0 \quad (x^2 + y^2 \to \infty),\\
&p(x,y,t) = -\phi_t(x,y,t),\\
&\frac{\partial^2 w}{\partial t^2} + \beta\frac{\partial^4 w}{\partial x^4} = p(x,0,t) \quad (|x| < \eta, t > 0),\\
&w = 0, w_{xx} \pm k w_x = 0 \quad (x = \pm\eta),\\
&w = w_t = 0 \quad (t = 0),\\
&\phi = \phi_t = 0 \quad (y < 0, t = 0).
\end{aligned}\right\} \tag{1}$$

Here $\dot{c}(t) > 1$ at the supersonic stage $0 < t < T$, $\dot{c}(T) = 1$, and $\dot{c}(t) < 1$ at the subsonic stage. The dot stands for the time derivative. The bending stress distribution $\sigma(x,t)$ is given in the dimensionless variables as $\sigma(x,t) = w_{xx}(x,t)$. In the non-dimensional variables the position of the body bottom is described by the equation

$$y = M(f(x) - t + \kappa w(x,t))$$

where $f(x)$ gives the initial shape of the bottom. In our case, $f(x) = x^2/2$. At the supersonic stage the free surface is still undisturbed, which leads to the equation $f(c) - t + kw(c,t) = 0$ for the function $c(t)$. At the subsonic stage $c(t)$ satisfies the equation derived by Korobkin (1992*a*). This transcendental equation follows from the additional condition that the displacements of the liquid particles are bounded.

The problem contains the four dimensionless parameters κ, β, η, k which are $\kappa = \rho_0 RM/M_B$, $\beta = EJ/(M_B R^2 V^2)$, $\eta = L/(RM)$ and k is the spring coefficient, where J is the inertia momentum of the beam cross-section. The rigid-body impact corresponds to the case $\kappa \ll 1$. When $\beta \ll 1$, we have the model of an inertial beam, and when $\eta \gg 1$, the model of an infinite beam. Different values of the spring coefficient k allow us to analyse the influence of different boundary conditions at the beam edges on the impact process. The values of the parameters are of great importance, because they determine what kind of problem simplifications can be used. For example, Kvålsvold & Faltinsen (1993*a*) in the numerical analysis of wetdeck slamming take $L = 75$ cm, $E = 7 \times 10^{10}$ N m^{-2}, $J = 1.106 \times 10^{-5}$m^3, $M_B = 36.6$ kg m^{-2}, $V = 6$ ms^{-1}, $c_0 = 1500$ ms^{-1}, $\rho_0 = 10^3$ kg m^{-3}, $R = 40$ m, $h = 12$ cm, $k = 3.5$. In this case

$$M = 4 \times 10^{-3}, \ \kappa = 4.37, \ \beta = 0.3672, \ \eta = 4.75 \tag{2}$$

and, hence, the coupled boundary-value problem (1) cannot be simplified. Moreover, the linear scale is 16 cm, the deflection scale is 2.3 mm, the time scale is 1.06×10^{-4}s, the stress scale is 918 MPa and the pressure scale is 9 MPa. These quantities mean that if we consider the beam as an undeformable rigid surface then the contact region size at the end of the supersonic stage is approximately 21% of the beam length and at the moment when the first left-hand-side relief wave escapes on the right-hand-side free surface it is 84% (see Korobkin 1992*a*).

However, any parallels between the present problem and the problem of an undeformable body impact are not clear, because owing to the beam response to the liquid impact it is possible that the acoustic stage of the process may be essentially reduced. We only suggest 'a possible reduction' because it is not clear in advance in what way the beam deflection affects the liquid motion. For example, in the model considered by Lesser (1981) taking account of the impacted surface elasticity leads to an increasing supersonic stage duration. This means that for different models of the elastic surface response the duration of the acoustic stage can be quite different. In order to clarify this point for the Euler beam model, we restrict ourselves to the supersonic stage only, when the free surface is still undisturbed and the boundary condition outside the contact region, $y = 0$, $|x| > c(t)$, can be changed for $\phi_y = 0$.

Let us determine the speed of the contact region expansion. At the supersonic stage ($0 < t < T$) the function $c(t)$ satisfies the equation

$$f(c) - t + \kappa w(c,t) = 0$$

which, after differentiation in time, gives

$$\frac{\mathrm{d}c}{\mathrm{d}t} = \frac{1 - \kappa w_t(c,t)}{f_x(c) + \kappa w_x(c,t)}. \tag{3}$$

From the physical point of view it is clear that $w_t(c,t) > 0$ and $w_x(c,t) < 0$. Hence, taking account of the elasticity of the impacted surface decreases the numerator and, at the same time, decreases the denominator of the right-hand side of (3). It is impossible to say in advance which effect – dynamical (the numerator decreasing) or geometrical (the denominator decreasing) – is more important. That is why the numerical investigation of the problem (1) is necessary.

3. Method of normal modes at the supersonic stage

In order to determine the pressure distribution over the contact region, the well-known formula (see Krasilshchikova 1982)

$$\phi(x,0,t) = \frac{1}{\pi} \iint_\sigma \frac{\phi_y(\xi,0,\tau)\mathrm{d}\xi\mathrm{d}\tau}{[(t-\tau)^2 - (x-\xi)^2]^{1/2}} \tag{4}$$

is used. The integration domain $\sigma(x,t)$ is shown in figure 3(a). The velocity potential is sought in the form

$$\phi(x,0,t)=\sum_{n=1}^{\infty} d_n(t)\psi_n(x) \tag{5}$$

on the part of the liquid surface where $|x|<\eta$, and the beam deflection $w(x,t)$ in the form

$$w(x,t)=\sum_{n=1}^{\infty} a_n(t)\psi_n(x). \tag{6}$$

Here $\psi_n(x)$ are the non-trivial solutions of the homogeneous boundary-value problem

$$\frac{d^4\psi_n}{dx^4}-\lambda_n^4\psi_n=0 \quad (|x|<\eta),$$

$$\frac{d^2\psi_n}{dx^2}\pm k\frac{d\psi_n}{dx}=0 \quad (x=\pm\eta)$$

where λ_n are the corresponding eigenvalues. Moreover, the eigenfunctions $\psi_n(x)$ satisfy the orthogonality condition

$$\int_{-\eta}^{\eta}\psi_n(x)\psi_m(x)dx=\delta_{nm},$$

where $\delta_{nm}=0$ when $n\neq m$ and $\delta_{nn}=1$. It is convenient to take the principal coordinates $a_n(t)$, $n=1,2,\ldots$ as the new unknown functions and to express with their help the coefficients $d_n(t)$ in (5).

At the stage $0<t<T$, the relation (4) can be rewritten as

$$\phi(x,0,t)=\frac{1}{\pi}\int_0^t\left(\int_{x+\tau-t}^{x+t-\tau}\frac{\phi_y(\xi,0,\tau)d\xi}{[(t-\tau)^2-(x-\xi)^2]^{1/2}}\right)d\tau$$

where $\phi_y(x,0,t)=(-1+\kappa w_t(x,t))H(c^2(t)-x^2)$, $H(x)=0$ for $x<0$ and $H(x)=1$ for $x\geqslant 0$. Hence,

$$d_n(t)=\frac{1}{\pi}\int_0^t\int_{-c(t)}^{c(t)}\psi_n(x)\left(\int_{x+\tau-t}^{x+t-\tau}\frac{\phi_y(\xi,0,\tau)d\xi}{[(t-\tau)^2-(x-\xi)^2]^{1/2}}\right)dxd\tau.$$

We shall now change the order of integration with respect to x and ξ. The region of integration on the plane (x,ξ) is shown in figure 3(b). In order to prove that this figure is right, one needs to prove the following inequalities:

$$\tau>t-c(t),\quad c(t)-c(\tau)>t-\tau.$$

Physical reasoning shows that the signal initiated at the initial moment at the centre point cannot reach the contact point during the supersonic stage. This means $c(t)>t$, which proves the first inequality because $\tau>0$. At the supersonic stage we have $\dot{c}(t)>1$. The integration of this relation in time from τ to t leads to the second inequality.

The additional change of integration variable

$$x=\xi+(t-\tau)\cos\theta$$

gives the final formula

$$d_n(t)=\frac{2}{\pi}\int_0^t\int_0^{c(\tau)}[-1+\kappa w_t(\xi,\tau)]\int_0^{\pi}\psi_n[\xi+(t-\tau)\cos\theta]d\theta d\xi d\tau. \tag{7}$$

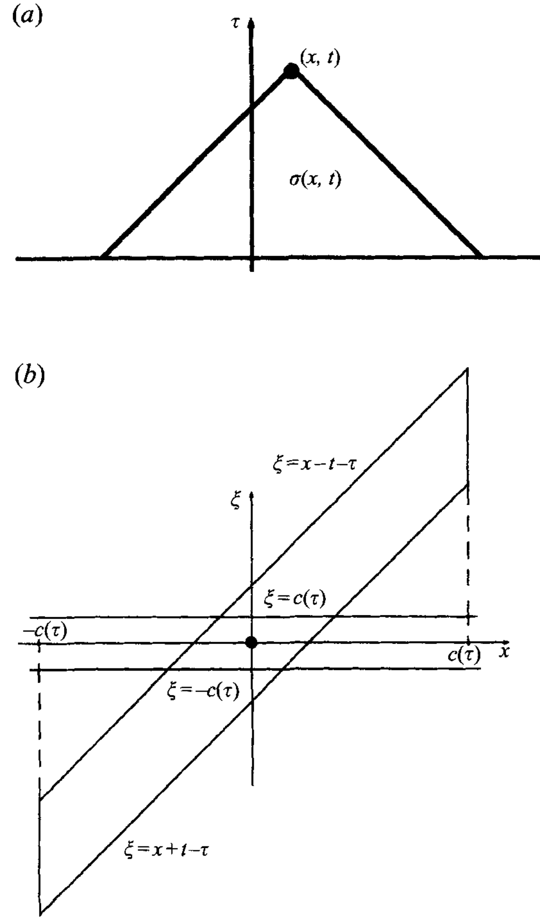

FIGURE 3. The geometry of the integration domain: (*a*) for determining the velocity potential; (*b*) to calculate the coefficients $d_n(t)$.

The pressure on the part of the liquid boundary where $|x| < \eta$ can be expressed as

$$p(x,0,t) = \sum_{n=1}^{\infty} p_n(t)\psi_n(x).$$

Here $p_n(t) = -\dot{d}_n(t)$, which follows from (5) and the linearized Cauchy–Lagrange integral $p = -\phi_t$. Taking (6), (7) into account, we obtain

$$p_n(t) = p_n^{(1)}(t) + p_n^{(2)}(t) + p_n^{(3)}(t).$$

Here

$$p_n^{(1)}(t) = \frac{2}{\pi}\int_0^t \int_0^{\pi} \cos\theta\psi_n[c(\tau) + (t-\tau)\cos\theta]\mathrm{d}\theta\mathrm{d}\tau + \int_{-c(t)}^{c(t)} \psi_n(x)\mathrm{d}x \tag{8}$$

is dependent only on $c(t)$ which itself depends on the unknown functions $a_n(t)$. The second term in (8) corresponds to the pressure under the impact at the moment t of a rigid plate, of width $2c(t)$, with a constant velocity on the undisturbed liquid surface. The first term describes the influence of the previous impact history on the pressure distribution at the moment t. The equation (8) is another form of the formula for the pressure distribution over the contact spot at the supersonic stage of the rigid-body impact (Korobkin 1992*b*).

The second term

$$p_n^{(2)} = -\kappa \sum_{m=1}^{\infty} \dot{a}_m(t) \int_{-c(t)}^{c(t)} \psi_n(x)\psi_m(x)\mathrm{d}x$$

is the sum of the pressures caused by the impacts of the rigid bodies, shapes of which correspond to the shapes of the eigenmodes of the beam deflection $\psi_n(x)$, at the moment t and over the region $-c(t) < x < c(t)$.

The coefficients $p_n^{(3)}(t)$ are

$$p_n^{(3)}(t) = -\frac{\kappa}{\pi} \sum_{m=1}^{\infty} \int_0^t \dot{a}_m(\tau)K_{nm}(t,\tau)\mathrm{d}\tau$$

where

$$K_{nm}(t,\tau) = \int_{-c(\tau)}^{c(\tau)} \psi_m(x) \int_0^{\pi} \cos\theta \psi_{nx}[x + (t-\tau)\cos\theta]\mathrm{d}\theta\mathrm{d}x.$$

It is worth noting that $K_{nm}(t,t) = 0$. This means that the functions $p_n^{(3)}(t)$ are dependent on the impact history and do not depend on any characteristics at the moment of observation t.

All integrals in the final expression for $p_n(t)$ can be evaluated analytically except for the integrals in time.

Substitution of (6) in the beam equation and the initial conditions gives

$$\left.\begin{aligned} \ddot{a}_n + \beta\lambda_n^4 a_n = p_n(t) \quad (t > 0), \\ a_n = 0, \quad \dot{a}_n = 0 \quad (t = 0). \end{aligned}\right\} \tag{9}$$

This infinite system of ordinary differential equations has to be supplemented by (3). Let us introduce the vector $\boldsymbol{Y}(t)$ of infinite length with components $Y_1 = c(t)$, $Y_2 = a_1$, $Y_3 = \dot{a}_1, \ldots, Y_{2N} = a_N$, $Y_{2N+1} = \dot{a}_N, \ldots$ Then the system (3), (9) can be written in the form

$$\boldsymbol{Y}_t = \boldsymbol{F}(\boldsymbol{Y}, t) \tag{10}$$

where $\boldsymbol{F}(\boldsymbol{Y}, t)$ is the nonlinear and non-local operator. The system (10) cannot be solved under the homogeneous initial conditions which follow from the original statement (1). Indeed, we get $F_1(0,0) = \infty$, $F_j(0,0) = 0$ $(j > 1)$ and, therefore, we are not able to start calculations. There are two possible ways to overcome this obstacle. The first approach, which is more exact, is to construct the initial asymptotics of the solution as $t \to 0$ (see Korobkin & Pukhnachov 1985) and to use it to start numerical calculations. The second approach is more practical and based on the assumption that we make a small alteration, changing the initial conditions for

$$Y_1 = c_r(\epsilon), \quad Y_j = 0 \ (j > 1) \quad (t = \epsilon) \tag{11}$$

where $\epsilon > 0, \epsilon \ll 1$, and the function $c_r(t)$ determines the position of the contact point in the rigid-body impact problem. For a parabolic shape of the bottom, $f(x) = x^2/2$, we have $c_r(t) = (2t)^{1/2}$ (see Bowden & Field 1964). The first approach also leads to (11) with Y_j being small but not equal to zero for $j > 1$. The problem (10), (11) corresponds to the case when initially $(t = \epsilon)$ the body touches the liquid surface over the region $-c_r(\epsilon) < x < c_r(\epsilon)$ and then starts to penetrate the liquid vertically.

The initial-value problem (10),(11) is numerically solved in the time interval (ϵ, T)

where T is unknown in advance and has to be determined using the conditions $F_1 > 1$ when $0 < t < T$, $F_1 = 1$ at $t = T$.

In order to prove that the method of normal modes can be applied to the problem (1), we have to estimate the error caused by the reduction of (10) to a finite system of differential equations. In this paper another approach is used. It is suggested that the asymptotic behaviour of the solution of the problem (9) as $n \to \infty$ is analysed using the following equalities:

$$\psi_n(x) = A_n \cos(\lambda_n x) + o(\lambda_n^{-\infty}) \text{ as } n \to \infty, |x| \leqslant c(t), 0 \leqslant t \leqslant T; \tag{12a}$$

$$A_n = \eta^{-1/2} + o(\lambda_n^{-\infty}), \lambda_n = \pi n/\eta + O(n^{-1}) \text{ as } n \to \infty; \tag{12b}$$

$$p(x,0,t) = [q_0(t) + q_1(t)(c^2(t) - x^2) + q_2(t)(c^2(t) - x^2)^2 + \cdots]H(c^2(t) - x^2)$$
$$\text{as } c^2(t) - x^2 \to 0, 0 \leqslant t < T; \tag{12c}$$

$$c(t) = (2t)^{1/2} + O(t) \text{ as } t \to 0. \tag{12d}$$

The asymptotic formulae (12*a*) and (12*b*) follow from the explicit forms of the eigenfunctions $\psi_n(x)$, the expansion (12*c*) from the general theory of water impact (see the Appendix), and the relation (12*d*) from the fact that $w(x,t) = O(t^2)$ as $t \to 0$ and the bottom can be considered as rigid just after the moment of impact. The asymptotic analysis of (9) will indicate how many eigenmodes have to be taken in (10) and will also clarify some characteristics of the beam deformation under the liquid impact.

The solution of (9) for every n has the form

$$a_n(t) = \frac{1}{\beta^{1/2}\lambda_n^2} \int_0^t p_n(\tau) \sin[\beta^{1/2}\lambda_n^2(t-\tau)]\mathrm{d}\tau \tag{13}$$

where

$$p_n(t) = \frac{2}{\lambda_n} A_n q_0(t) \sin[\lambda_n c(t)] + O(n^{-2})$$

for $n \to \infty$, which follows from the asymptotic formula (12*a*) and the expansion (12*c*) Integrating (13) by parts, we obtain

$$a_n(t) = \frac{1}{\beta\lambda_n^4} p_n(t) - \frac{1}{\beta\lambda_n^4} \int_0^t \dot{p}_n(\tau) \cos[\beta^{1/2}\lambda_n^2(t-\tau)]\mathrm{d}\tau. \tag{14}$$

The asymptotic behaviour of the integral in (14) as $n \to \infty$ is determined by the singularity of $\dot{p}_n(t)$ as $t \to 0$. Taking into account the asymptotic formula (12*d*) and the equality $q_0(0) = 1$ (see the Appendix), we obtain

$$\dot{p}_n(t) = \frac{\sqrt{2}}{t^{1/2}} A_n \cos[(2t)^{1/2}\lambda_n] + O(n^{-1})$$

as $t \to 0$ and $n \to \infty$. Therefore, for large n

$$a_n(t) = \frac{1}{\beta\lambda_n^4} p_n(t) - \frac{2A_n}{\beta\lambda_n^5}[\cos(\beta^{1/2}\lambda_n^2 t)S_1(\beta) + \sin(\beta^{1/2}\lambda_n^2 t)S_2(\beta)] + o(\lambda_n^{-5})$$

where

$$S_1(\beta) = \int_0^\infty \cos\sigma \cos(\tfrac{1}{2}\beta^{1/2}\sigma^2)\mathrm{d}\sigma, \quad S_2(\beta) = \int_0^\infty \cos\sigma \sin(\tfrac{1}{2}\beta^{1/2}\sigma^2)\mathrm{d}\sigma.$$

The final result is as follows: the principal coordinates $a_n(t)$ are of $O(n^{-5})$ at the supersonic stage as $n \to \infty$, the functions $n^5 a_n(t)$ therewith oscillate with the frequency $\beta^{1/2}\lambda_n^2$. These oscillations are due to the initial strong discontinuity of the hydrodynamic load and are not affected at the leading order by the subsequent evolution of the beam deformation and the load variations.

It is worth noting that for the coupled problem of the beam–liquid impact (1) the contribution of the oscillations will be the same as derived above. This is due to the fact that these oscillations are generated at the moment of first impact when the beam deflection is negligible and are independent of the process evolution.

The last remark follows from the beam model which does not involve any mechanism for energy dissipation. Therefore, characteristics of the deflection which appear at some time instant will appear and can be indicated for every following times. The contribution of these oscillations can be separated from the series of normal modes and analysed in detail.

4. Numerical results

The initial value problem (10), (11) was solved numerically by the Runge–Kutta method with step 0.001; the integrals in time were evaluated by Simpson's rule with the same fixed step. The number of eigenmodes N_{mod} was limited to 20, the quantity ϵ being taken as 0.0011. It was verified that the final results are independent of small variations of ϵ.

In the case given in (2) that corresponds to the numerical analysis of the wetdeck-wave impact problem by Kvålsvold & Faltinsen (1993*a*) the duration of the supersonic stage T was found to be 0.32, in contrast to the rigid-body impact where $T = 0.5$. With increasing the parameter κ the value of T decreases. For example, $T = 0.238$ for $\kappa = 10$ and $T = 0.4351$ when $\kappa = 1$. This means that the reduction of the impact velocity due to the beam deformation is a more important factor than the variation of the beam geometry.

The beam deflection at $t = T$ is shown in figure 4(*a*), the size of the contact region being 1.5 in the non-dimensional variables. There are two curves plotted, for $N_{mod} = 20$ and 15, but it is quite difficult to find any difference between them. The beam shapes for different values of κ at the moment $t = 0.2$ are shown in figure 4(*b*). In order to estimate the influence of the parameter on the deflection amplitude, one has to multiply the dimensionless values presented in this figure by the deflection scale (see §2) and to consider different reasons of the variation of κ. If this variation is caused by the change of the beam mass M_B then the coefficient for the variation is equal to the ratio of these masses. If it is due to the change of the initial impact velocity V then the coefficient is equal to the cubic of the impact velocity ratio. In any case it can be found that the deflection amplitude increases with the increasing κ.

In the case given in (2) the amplitude of the beam deflections is quite small, approximately 0.5 mm. Nevertheless, the flexibility of the beam cannot be neglected. In order to demonstrate this point, let us consider the vertical velocity of the beam elements (figure 5). We can estimate that to the moment $t = 0.2$ the impact velocity has been halved due to the beam deflection. The variation of the impact velocity leads to the reduction of the hydrodynamic force (figure 6). It is of importance that in the Euler beam model the total force is not monotonic in time at the supersonic stage. This conclusion agrees with the results by Kvålsvold & Faltinsen (1993*a*). The maximum value of the force at this stage is reached at the moment $t' = 2 \times 10^{-5}$s and is equal approximately to 10^6 N m^{-1}. On the other hand, the reduction of the size of the contact region due to the body elasticity is less than 7.5% (see figure 7). This means that the main effect of an elastic body impact on the liquid is more dynamical than geometrical.

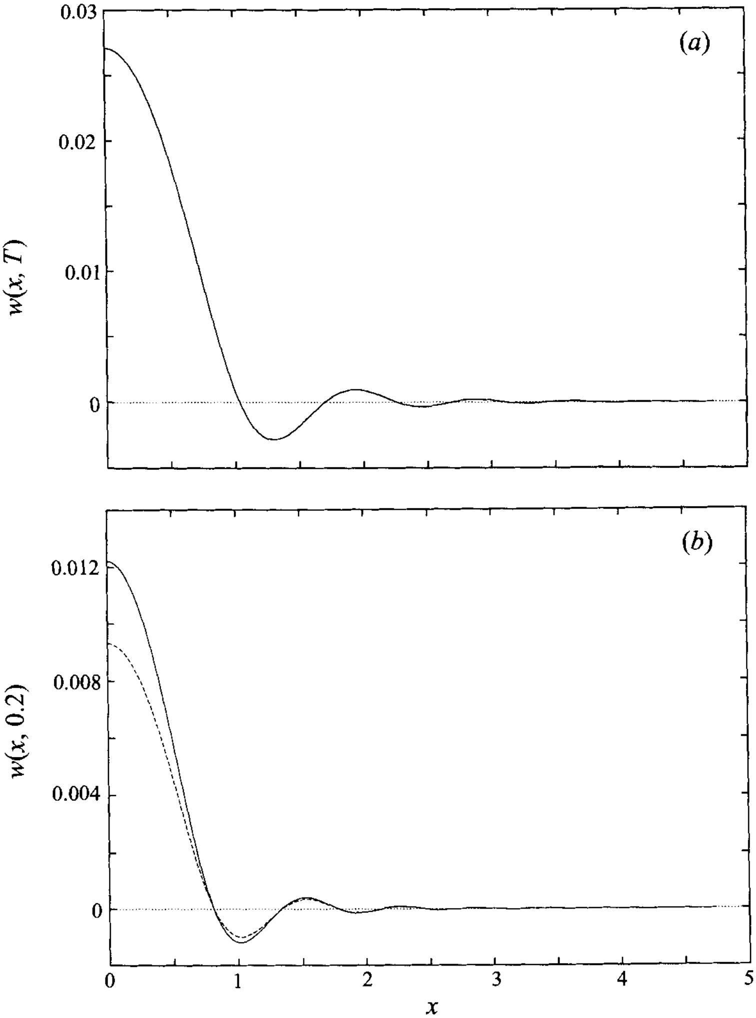

FIGURE 4. The beam deflection $w(x,t)$ as a function of x: (a) at the end of the supersonic stage (non-dimensional variables); (b) at the moment $t = 0.2$ for ——, $\kappa = 4.37$; – – –, $\kappa = 10$ (non-dimensional variables).

The bending stress distribution along the beam at the moment $t = T$ is shown in figure 8. The magnitude of the bending stresses corresponds to that reported by Kvålsvold & Faltinsen (1993a). It is seen that correct calculations of the dynamical characteristics need more normal modes than for the geometrical ones. The stresses are high and positive just in front of the contact point, and are negative over the main part of the contact region. In the dimensional variables at the end of the supersonic stage, $t' = 3.41 \times 10^{-5}$s, the stress at the centre point is -69 MPa and it takes its maximal value, which is 83 MPa, at $x' = 17.5$ cm. Half the contact region width at this moment is 12 cm. We can conclude that the back side of the beam is expanded in the contact region due to the beam deformation and is compressed in front of the contact point. The last effect is shown to be quite strong.

The bending stress distributions at the moment $t = 0.2$ are shown in figure 9 for three different values of κ. It is seen that the variation of this parameter does not

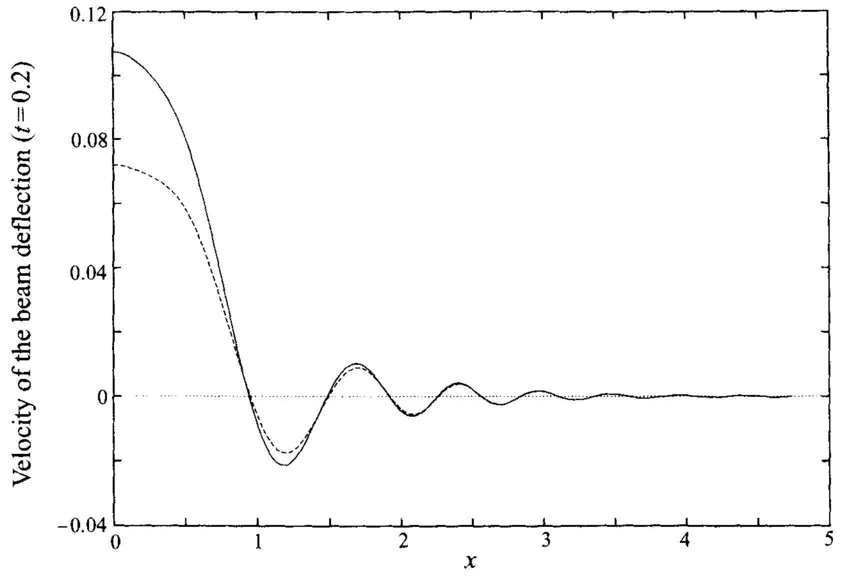

FIGURE 5. The vertical velocity of the beam elements at $t = 0.2$. ——, $\kappa = 4.37$; – – –, $\kappa = 10$ (non-dimensional variables).

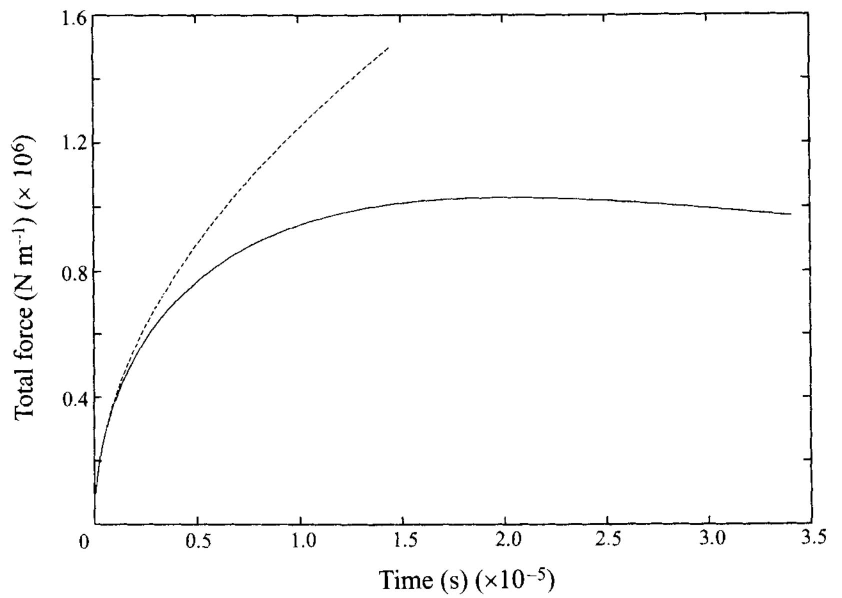

FIGURE 6. The hydrodynamic force on the entering body. ——, $\kappa = 4.37$; – – –, $\kappa = 0$ (rigid body).

change the structure of the distribution but only the stress amplitude. The evolution of the maximum value of the bending stress and the location of this peak as a function of time are shown in figures 10(*a*) and 10(*b*) respectively. The initial part of the last figure is expected because the initial dimension of the contact region is not zero but equals approximately 2 cm in the present numerical calculations. To clarify this statement, the self-similar solution mentioned in §1 has to be considered.

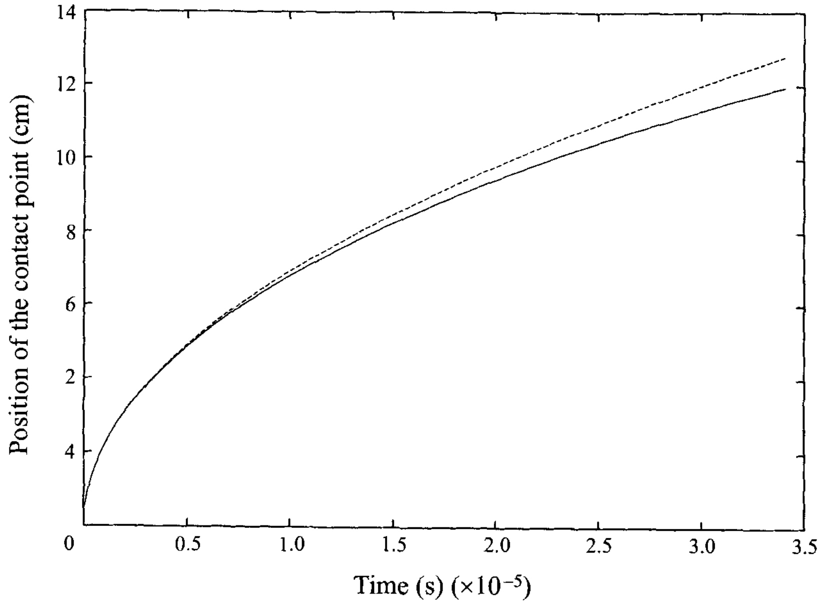

FIGURE 7. Position of the contact point. ——, $\kappa = 4.37$; – – –, $\kappa = 0$ (rigid body). Half the beam width is 75 cm.

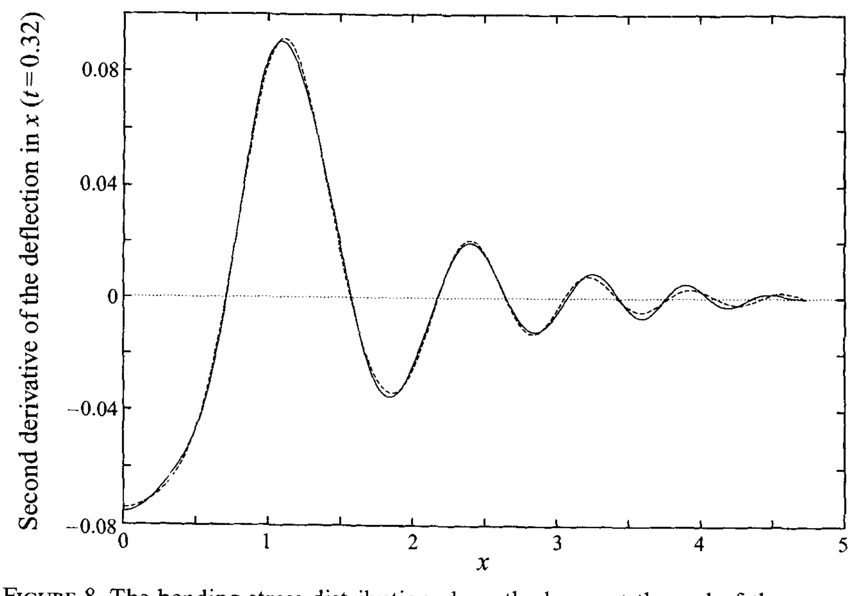

FIGURE 8. The bending stress distribution along the beam at the end of the supersonic stage, $t = 0.32$, $\kappa = 4.37$. ——, $N_{mod} = 20$; – – –, $N_{mod} = 15$ (non-dimensional variables).

5. The pressure distribution over the wetted part of the beam

The pressure distribution over the contact region cannot easily be determined by the normal mode method owing to the strong discontinuity of the pressure at the contact points. Therefore, we have to calculate this pressure directly using the general theory (see Korobkin 1992*b*). On the other hand, we can improve the convergence of the mode series for the pressure in the same manner as it was done by Korobkin (1996) and use the series at least to estimate the pressure variation on the contact

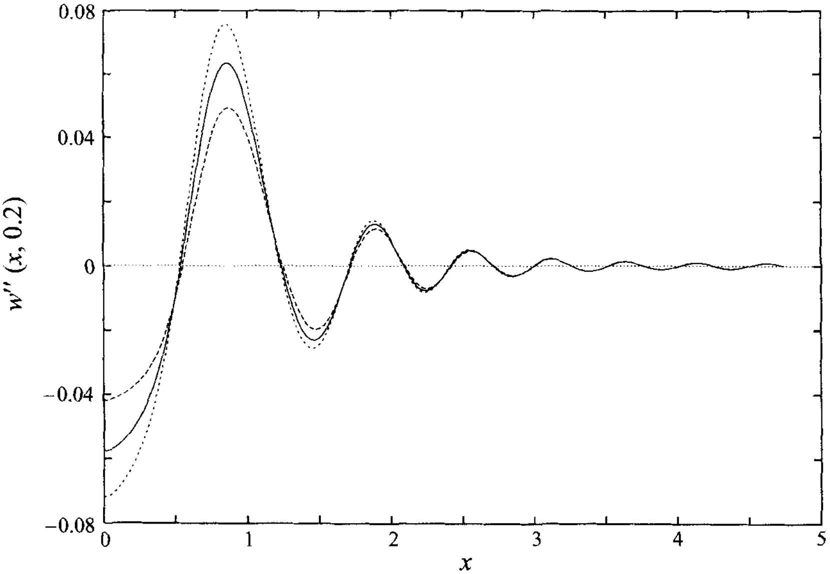

FIGURE 9. The bending stress distribution at the moment $t = 0.2$. ——, $\kappa = 4.37$; – – –, $\kappa = 10$; - - - -, $\kappa = 1$ (non-dimensional variables).

spot. In order to improve the convergence, we write (see the Appendix)

$$p(x,0,t) = p_c(t)H[c^2(t) - x^2] + P(x,t) \tag{15}$$

where $p_c(t)$ is the pressure at the contact point, $H(x) = 1$ where $x > 0$ and $H(x) = 0$ where $x < 0$, and $P(x,t)$ is the regular part of the pressure, $P(x,t) = O(x^2 - c^2(t))$ as $x^2 \to c^2(t) - 0$. We know that $p_n(t) = O(n^{-1})$ but $P_n(t) = O(n^{-2})$ as $n \to \infty$. Here

$$P_n(t) = \int_{-\eta}^{\eta} P(x,t)\psi_n(x)\mathrm{d}x.$$

Multiplying the both sides of (15) by $\psi_n(x)$ and integrating in x from $x = -\eta$ to $x = +\eta$, one gets

$$P_n(t) = p_n(t) - p_c(t)\int_{-c(t)}^{c(t)} \psi_n(x)\mathrm{d}x. \tag{16}$$

The coefficients $p_n(t)$ were found by the solving the system (10). The function $p_c(t)$ can be determined in the same manner as for a rigid-body impact (see Korobkin 1992*b* and the Appendix) and is

$$p_c(t) = (1 - \kappa w_t[c(t),t])\frac{\dot{c}(t)}{([\dot{c}(t)]^2 - 1)^{1/2}}. \tag{17}$$

Using (16), we can evaluate $P(x,t)$ quite correctly and then calculate the pressure on the contact spot with the help of (15). This procedure fails as $t \to T - 0$ because the pressure behaviour changes radically at the end of the supersonic stage and is no longer determined by (15).

6. Wave impact on a beam

Comparing the present results with those by Kvålsvold & Faltinsen (1993*a*), we may say that the acoustic approximation describes the initial stage of the impact more

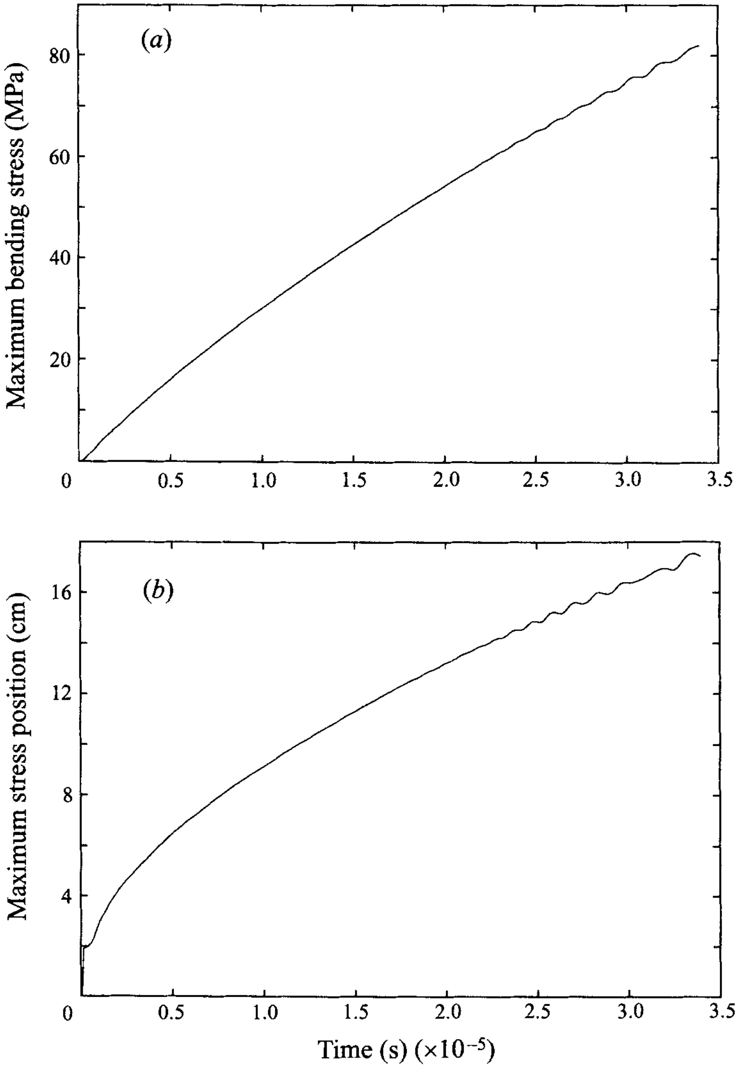

FIGURE 10. (*a*) The maximum bending stress as function of time, $\kappa = 4.37$, and (*b*) its position.

accurately but is only expected to have a slight influence during the later stages of the interaction when compressibility can be neglected. That is why it is suggested that the dependence of the solution of (1) on the parameters κ, β, η should be analysed and their values for which the acoustic effects are of major significance should be found.

We will consider the problem of wave impact onto the wetdeck of a catamaran (see figure 2). The wave is assumed linear and its shape given by the equation

$$y = a \sin(\nu x - \omega t)$$

where a is the wave amplitude, ω is the wave frequency, and ν is the wavenumber. The radius of the curvature R at the top of the wave profile is equal now to

$$R = (a\nu^2)^{-1}.$$

For a deep water, $\omega = (g\nu)^{1/2}$, we get

$$R = (2\pi)^{-4} g^2 T_w^4 a^{-1} \tag{18}$$

where g is the gravity acceleration and T_w is the wave period. The parameter values in (2) correspond to the case $V = 6\ \mathrm{m\,s^{-1}}$, $a = 2$ m, $T_w = 6$ s. The definitions of κ, β, η demonstrate that when the wave characteristics a, T_w and the impact velocity V only are varying, the solution is not changed if the product RV remains constant. This means that for a given wetdeck any two impact processes will be mechanically similar if the corresponding dimensionless parameters η_1 and η_2 are the same. Using (18), this similarity criterion can be written in the form

$$\frac{V_1 T_{w1}^4}{a_1} = \frac{V_2 T_{w2}^4}{a_2}.$$

Hence, the characteristics of the impact process depend on the wave period rather than on both the wave amplitude and the impact velocity.

Let us consider the impact on the catamaran wetdeck at velocity $V = 4\ \mathrm{m\,s^{-1}}$ of a wave of amplitude $a = 2$ m and of period $T_w = 10$ s and show that this case differs from that analysed above. Now we get

$$R = 308.7\ \mathrm{m}, \quad M = 2.6 \times 10^{-3}, \quad \eta = 0.91, \quad \kappa = 22.5, \quad \beta = 0.014. \tag{19}$$

Hence, in this case the contact points will be close to the beam edges at the end of the supersonic stage during which the acoustic effects in combination with the elastic ones are of major importance.

For the parameters in (19), the evolution of the pressure at the contact point (17) is shown in figure 11. In contrast, for the rigid-body impact $p_c(t)$ increases monotonically in time. This figure demonstrates once again that any analogies between rigid-body impact and elastic-body impact are quite limited. The evolution of the hydrodynamic force is shown in figure 12. It is seen that the elastic effects start to reduce the force quite early after the impact moment, but the force behaviour is similar to that presented in figure 6. The variation of the bending stresses with time at the point of first impact, $x = 0$, is shown in figure 13. The oscillations are due to the beam edges influence. It should be noted that the stresses are quite small, less than 30 MPa. On the other hand, the bending moments in front of the moving contact point (see figure 14) can be quite large (see figure 15). It can be supposed that the maximum stress value is dependent on the duration of the impact rather than on its intensivity.

Preliminary calculations based on (16) have shown that the pressure can become negative quite early after the impact moment, $t = 0$, but the total force remains positive at the same time (figure 16). The effect of the pressure drop due to the elasticity of the entering surface is strong and can be observed in three-dimensional drop tests as well (Zhu & Faulkner 1994). This effect can be explained using the analogy of the impact between the railway wagons. This means that cavitation can occur just after the instant of first impact.

7. Conclusion

It is demonstrated in the present paper that the normal mode method is a powerful tool to treat the problem of elastic body impact onto the free surface of an ideal and weakly compressible liquid. This method has to be applied quite carefully because it does not take into account any characteristics of the solution of the impact problem. Nevertheless, for the Euler beam model the method is reasonable. It was shown that

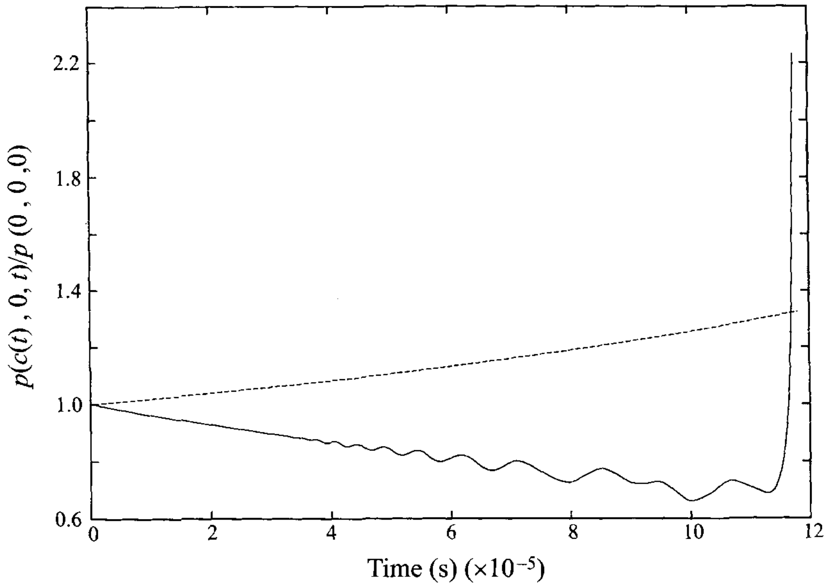

FIGURE 11. The evolution of the pressure at the contact point at the supersonic stage. ——, $\kappa = 22.5$, $\beta = 0.014$, $\eta = 0.91$, $M = 2.6 \times 10^{-3}$; – – –, $\kappa = 0$ (rigid body).

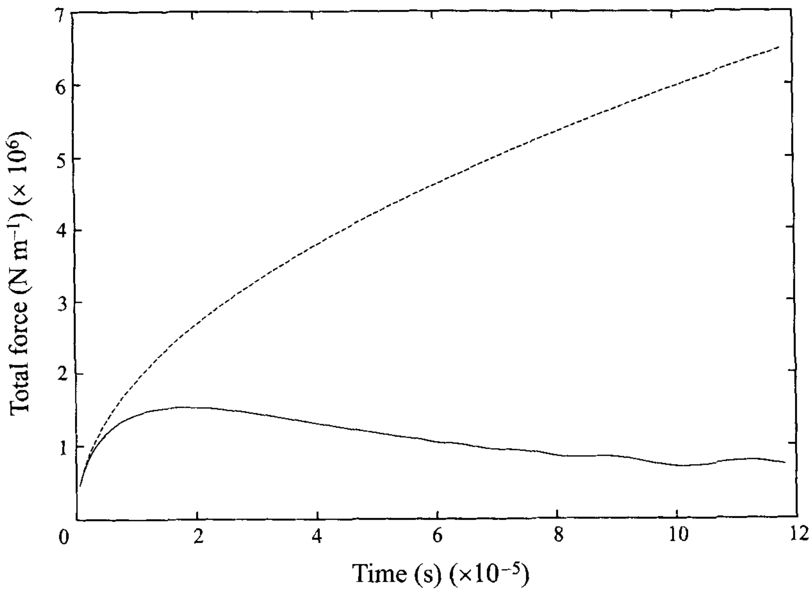

FIGURE 12. The hydrodynamic force on the entering body. ——, $\kappa = 22.5$; – – –, $\kappa = 0$ (rigid body).

the acoustic part of the impact history is of importance for long waves. This part is where the hydrodynamic force reaches its maximal value at the initial stage.

In §6 the criterion which demonstrates the respective influence of the wave amplitude, wave period and the impact velocity on the wave impact was derived.

The pressure on the contact spot can become negative at the supersonic stage, and separation of the liquid from the beam can be expected. Cavitation under the impact of an elastic body with a liquid may radically change the pressure distribution over

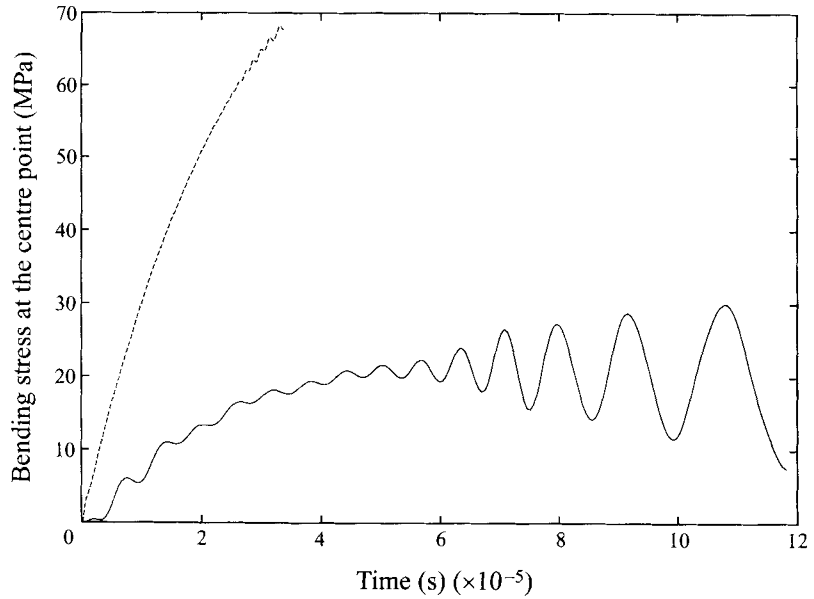

FIGURE 13. The bending stresses at the point of the first contact, $x = 0$. ——, $\kappa = 22.5$; – – –, $\kappa = 4.37$.

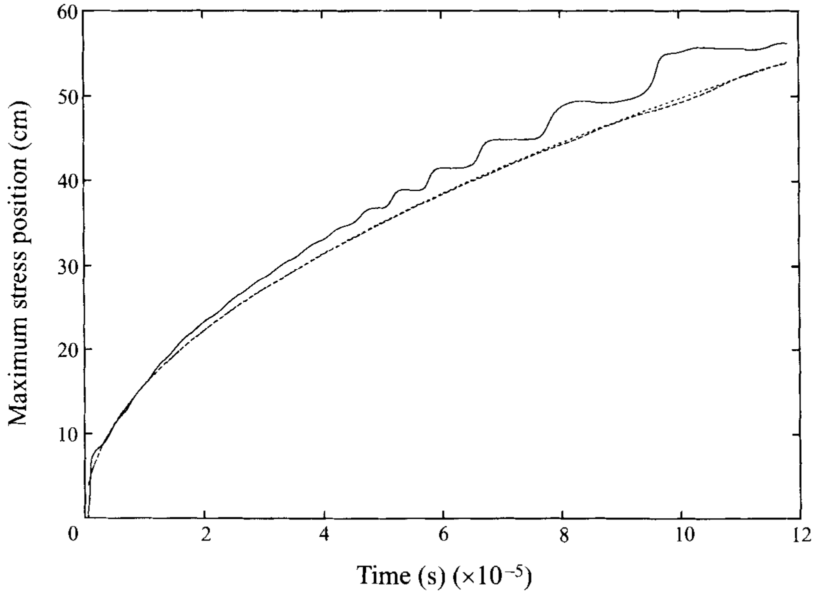

FIGURE 14. The position of the maximum bending stress as function of time (——); the position of the contact point (– – –) for $\kappa = 22.5$; the position of the contact point for the rigid body (- - - -).

the contact region and may lead to erosion of the impacting surface. The separation of the liquid from the impacting surface can be treated in the same manner as for rigid-body impact (see Korobkin 1994) but now the nature of the cavitation is not acoustical but inertial.

It is of importance to continue the present analysis to describe the subsonic stage of the impact after the escape of the shock wave onto the free surface. The only

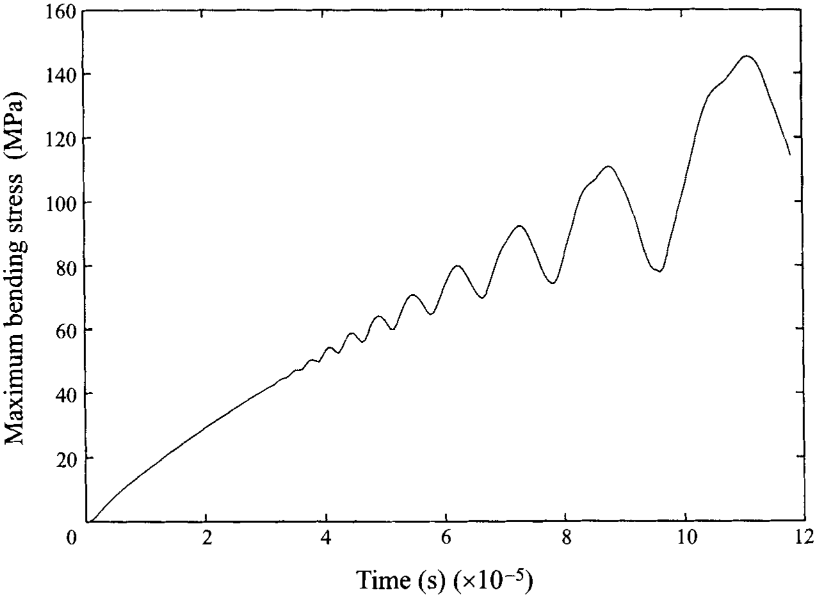

FIGURE 15. The maximum bending stress as function of time, $\kappa = 22.5$.

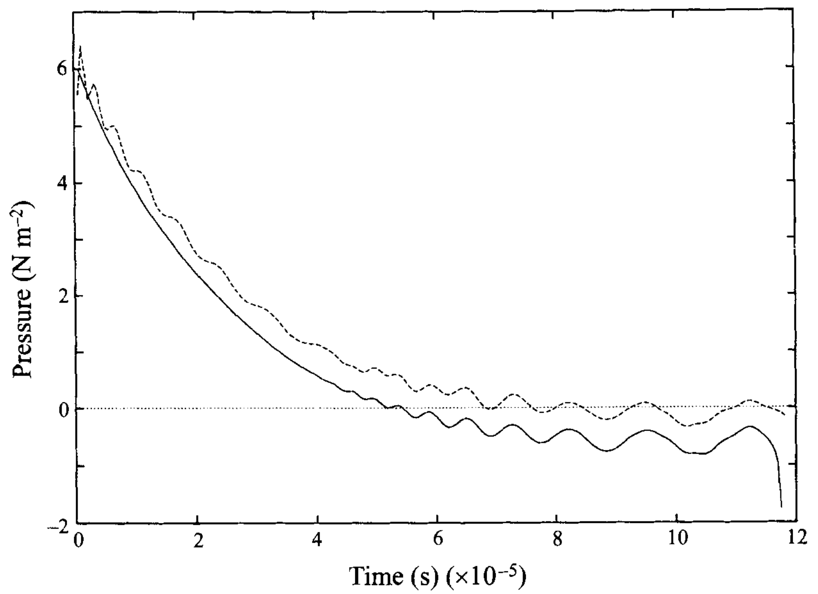

FIGURE 16. The pressure at the point of the first contact, $x = 0$, as function of time. ——, modified method of normal modes; – – –, method of normal modes.

obstacle here is deriving the formulae for the pressure coefficients $p_n(t)$, which are quite complicated.

It is desirable to change the initial conditions (11) for those which correspond to the original statement (1). This can be done with the help of the self-similar solutions of the beam equation (see §1). It is not difficult to present those solutions in quadratures, but the subsequent analysis is complicated and will be given in a future paper.

The method of calculation of the pressure on the contact spot, which was presented above, can be used only for estimations and a more accurate method is needed.

It is important to investigate the impact problem for more general models of an elastic surface (see Kvålsvold & Faltinsen 1994). It is of interest to note that within the framework of the Timoshenko beam model the beam deformation is governed by a hyperbolic system of differential equations. The system has the characteristic velocities of signal propagation for the deflections and the shear angle. Those velocities are comparable with the sound speed in the resting water for the case considered by Kvålsvold & Faltinsen (1994). Therefore, one can expect that at the stages when the velocity of the hydrodynamic load expansion along the beam is close to the characteristic ones, the stresses in the beam may be singular and, hence, the cumulation effect may be observed. This means that at these stages much greater stresses can be expected that predicted by the Euler beam model.

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Appendix. Asymptotic behaviour of the pressure near the contact points

At the supersonic stage the velocity potential (see §3) in the contact region, $|x| < c(t)$, $y = 0$, is

$$\phi(x,0,t) = \frac{1}{\pi}\int_0^t \left(\int_{x+\tau-t}^{x+t-\tau} \frac{[-1+\kappa w_t(\xi,\tau)]H[c^2(\tau)-\xi^2]\,d\xi}{[(t-\tau)^2-(x-\xi)^2]^{1/2}}\right) d\tau,$$

the integration domain ARL is shown in figure 17. Owing to the flow symmetry, the pressure behaviour near the right-hand contact point, $x = c(t), 0 < t < T$, only is considered. The coordinates of the points A, R and L are $(x,t), (\xi_R, \tau_R)$ and (ξ_L, τ_L), respectively. The functions $\xi_R, \xi_L, \tau_R, \tau_L$ satisfy the equations

$$\left.\begin{aligned} \tau_L &= c(\tau_L) + t - x, \quad & \xi_L &= c(\tau_L), \\ \tau_R &= -c(\tau_R) + x + t, \quad & \xi_R &= c(\tau_R), \end{aligned}\right\} \tag{A 1}$$

and depend on x and t. It is convenient to denote the local velocity $-1 + \kappa w_t(x,t)$ by $V(x,t)$ and split the integration domain into two parts (see figure 17)

$$\phi(x,0,t) = \frac{1}{\pi}\left(\int_{\tau_L}^{\tau_R}\int_{x+\tau-t}^{c(\tau)} + \int_{\tau_R}^{t}\int_{x+\tau-t}^{x+t-\tau}\right) \frac{V(\xi,\tau)\,d\xi\,d\tau}{[(t-\tau)^2-(x-\xi)^2]^{1/2}}.$$

An additional change of the integration variable

$$\xi = x + (t-\tau)\sigma$$

gives the formula

$$\phi(x,0,t) = \frac{1}{\pi}\left[\int_{\tau_L}^{\tau_R}\int_{-1}^{\frac{c(\tau)-x}{t-\tau}} + \int_{\tau_R}^{t}\int_{-1}^{1}\right] \frac{V(x+(t-\tau)\sigma,\tau)\,d\sigma\,d\tau}{(1-\sigma^2)^{1/2}},$$

which is suitable for differentiation with respect to time. The linearized Cauchy–Lagrange integral $p = -\phi_t$ makes it possible to determine the pressure distribution

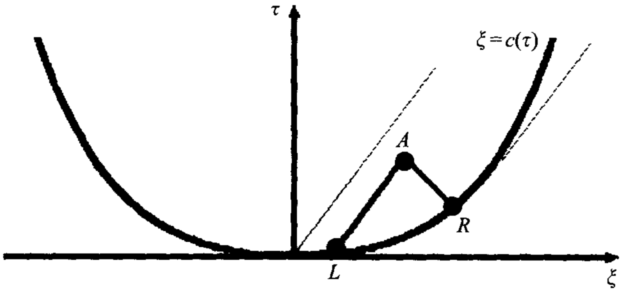

FIGURE 17. The integration domain.

near the right-hand contact point as

$$p(x,0,t) = -V(x,t) + \frac{1}{\pi}\int_{\tau_L}^{\tau_R} \frac{V[c(\tau),\tau]}{[(t-\tau)^2-(c(\tau)-x)^2]^{1/2}}\frac{c(\tau)-x}{t-\tau}\,d\tau$$
$$-\frac{1}{\pi}\left(\int_{\tau_L}^{\tau_R}\int_{-1}^{\frac{c(\tau)-x}{t-\tau}} + \int_{\tau_R}^{t}\int_{-1}^{1}\right)\frac{V_x(x+(t-\tau)\sigma,\tau)\sigma\,d\sigma}{(1-\sigma^2)^{1/2}}\,d\tau. \quad \text{(A 2)}$$

This formula is valid when $0 < t < T, t < x < c(t)$, and it can be used to evaluate the pressure distribution numerically.

In order to find the asymptotic behaviour of the pressure close to the contact point, $x \to c(t) - 0$, the 'internal' variable $\epsilon = c(t) - x$ is introduced. We shall determine two first terms in the asymptotic expansion of $p(c(t)-\epsilon, 0, t)$ as $\epsilon \to 0$ under the assumption that the first and second derivatives of the function $V(x,t)$ are bounded.

The first term in (A2) gives

$$\Gamma_1 := V[c(t)-\epsilon, t] = V[c(t),t] - V_x[c(t),t]\epsilon + O(\epsilon^2). \quad \text{(A 3)}$$

With the help of the change of the integration variable

$$\frac{c(\tau)-x}{t-\tau} = s \quad \text{(A 4)}$$

the second term in (A2) can be written in the form

$$\Gamma_2 := \frac{1}{\pi}\int_{-1}^{1}\frac{V[c(\tau(s,x,t)),\tau(s,x,t)]s\,ds}{(\dot c(\tau(s,x,t))+s)(1-s^2)^{1/2}}, \quad \text{(A 5)}$$

which is more suitable for asymptotic analysis. Figure 17 shows that $\tau = t - \tau_1$ where $\tau_1 \to 0$ as $\epsilon \to 0$. Equation (A4) yields

$$\tau_1 = \frac{\epsilon}{s+\dot c(t)} + O(\epsilon^2).$$

Therefore

$$c(\tau) = c(t) - \frac{\dot c(t)}{\dot c(t)+s}\epsilon + O(\epsilon^2),$$
$$\dot c(\tau) = \dot c(t) - \frac{\ddot c(t)}{\dot c(t)+s}\epsilon + O(\epsilon^2),$$
$$\frac{1}{\dot c(\tau)+s} = \frac{1}{\dot c(t)+s} + \frac{\ddot c(t)}{[\dot c(t)+s]^3}\epsilon + O(\epsilon^2),$$
$$V[c(\tau),\tau] = V[c(t),t] - \left\{V_x[c(t),t]\frac{\dot c(t)}{\dot c(t)+s} + V_t[c(t),t]\frac{1}{\dot c(t)+s}\right\}\epsilon + O(\epsilon^2),$$

which gives the asymptotic expansion of the integral (A5) in the form

$$\begin{aligned}\Gamma_2 := \frac{1}{\pi}V[c(t),t]\int_{-1}^{1}\frac{s\,\mathrm{d}s}{[\dot{c}(t)+s](1-s^2)^{1/2}}\\ +\frac{\epsilon}{\pi}\left\{V[c(t),t]\ddot{c}(t)\int_{-1}^{1}\frac{s\,\mathrm{d}s}{[\dot{c}(t)+s]^3(1-s^2)^{1/2}}\right.\\ \left.-\frac{\mathrm{d}}{\mathrm{d}t}(V[c(t),t])\int_{-1}^{1}\frac{s\,\mathrm{d}s}{[\dot{c}(t)+s]^2(1-s^2)^{1/2}}\right\}+O(\epsilon^2).\end{aligned} \tag{A 6}$$

The inner integral in the third term in (A2) is bounded, and $\tau_R \to t-0, \tau_L \to t-0$ as $\epsilon \to 0$. Therefore, this term is of $o(1)$ as $\epsilon \to 0$. Equations (A1) predict

$$\tau_L = t - \frac{\epsilon}{\dot{c}(t)-1} + O(\epsilon^2),$$

$$\tau_R = t - \frac{\epsilon}{\dot{c}(t)+1} + O(\epsilon^2).$$

Using the new integration variable $\tau = t - \epsilon s$, we obtain the asymptotic expansion of the third term in (A2) when $\epsilon \to 0$ as

$$\Gamma_3 := -\epsilon\frac{1}{\pi}V_x[c(t),t]\int_{1/(1+\dot{c}(t))}^{1/(\dot{c}(t)-1)}\mathrm{d}s\int_{-1}^{1/s-\dot{c}(t)}\frac{\sigma\,\mathrm{d}\sigma}{(1-\sigma^2)^{1/2}}+O(\epsilon^2)$$

and after simplifications

$$\Gamma_3 = \epsilon\frac{1}{\pi}V_x[c(t),t]\int_{\dot{c}(t)-1}^{\dot{c}(t)+1}[1-(u-\dot{c})^2]^{1/2}\frac{\mathrm{d}u}{u^2}+O(\epsilon^2). \tag{A 7}$$

The fourth term in (A2) is of $O(\epsilon^2)$ as $\epsilon \to 0$. Indeed, the dimension of the interval of integration in the outer integral with respect to τ is $t - \tau_R = O(\epsilon)$ and the inner integral is equal to zero when $\tau = t$.

We obtain

$$p(x,0,t) = p_c(t) + A(t)(c(t)-x) + O[(c(t)-x)^2]$$

as $c(t) - x \to 0$. The formulae for $p_c(t)$ and $A(t)$ follow from (A3), (A6) and (A7). In particular,

$$p_c(t) = -V[c(t),t]\frac{\dot{c}(t)}{(\dot{c}^2(t)-1)^{1/2}}.$$

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